ASIAN JOURNAL OF CIVIL ENGINEERING (BUILDING AND HOUSING) VOL.1, NO.2, (2000)
PAGES 83-88

TECHNICAL NOTE

MODIFIED ODE-SOLVER FOR BUCKLING OF COUPLED SHEAR-WALL BUILDINGS

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ABSTRACT

The main purpose of this paper is to present a numerical solution for buckling of coupled shear-wall tall buildings by using a modified ordinary differential equations (ODE_S) solver. In order that a general purpose ODE solver may be compatible with solving the eigenvalue problem, a solution strategy for general purpose ODE solver is modified. That is, the strategy organically puts the following techniques together: "Standard" transform for nonlinear problem; Iteration of reciprocal power; Orthogonalization and Eigenvalue displacement et al. In comparison with the results calculated by Galerkin's method of weighted residuals shows that existing ODE solver can conveniently be taken for accurate and reliable solution of the eigenvalue problem of stability.

Keywords: ODE-solver, buckling, coupled shear-wall, weighted residuals

1. INTRODUCTION

The rapid advances of new computational technology for solution of ordinary differential equation (ODE) in boundary value problems have made a number of high quality general purpose ODE solvers available, however, various ODE problems arising in engineering application are frequently not in the "standard" form for these codes. However, many problems can be transformed into such forms by using some simple and novel conversion techniques. In this paper, a solution strategy for eigenvalue of overall stability of coupled shear-wall tall buildings is presented by using modified ODE solver. The complex buckling problem is also solved effectively.

2. ODE FOR BUCKLING OF COUPLED SHEAR-WALL

A coupled shear-wall with constant properties throughout the whole structural height, **H**, is considered, as shown in Figure 1. When a continuous, incompressible medium for the coupling beams is assumed, the differential equation governing the buckling problem of the coupled shear-wall is given by (see reference [1]):

$$\phi''''(\xi) - a^2H^2(1+\gamma)''(\phi) = -\lambda[\xi\phi'(\xi) + 2\phi' - a^2H^2\gamma\xi\phi(\xi)] \quad (1)$$

in which $\varphi = 1\text{-}z/H$; $\phi(\varphi)=y'(\varphi)$; $y(\varphi)$ is the buckling mode of the structure;

$$\alpha^2 = 12 I_{cb} l^2/(ha^3 I); r = I/I_n;$$
$$I_n = A_1 A_2 l^2 / A;$$
$$A = A_1 + A_2;$$
$$I = I_1 + I_2;$$
$$I_{cb} = I_b /[1+12EI_b /(a^2GA_b)];$$
$$\lambda = q_{cr} H^3 /(EI); \quad (2)$$

I_i and I_b are the moments of inertia of each wall and the connecting beam, respectively; A_i is the cross section of each wall; A_b is the effective shear area of the connecting beam; **E**, and **G** are the Young's and shearing modulus of elasticity, respectively; **l** is the distance between the axes of coupled wall; **h** is the story height; a is the width of opening, and q_{cr} is the critical uniformly distributed axial load.

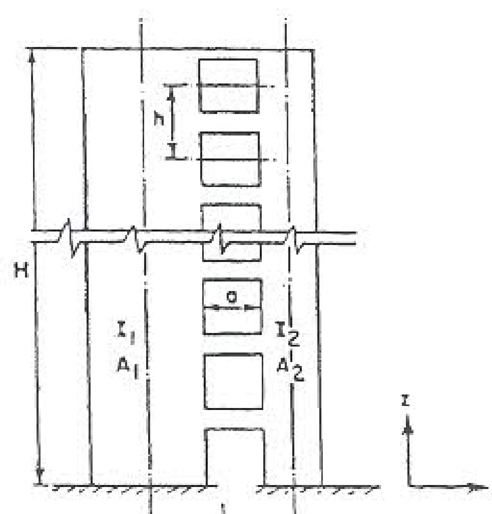

Figure 1 Coupled shear-wall

The corresponding boundary conditions are

$$\phi'(0) = \phi'''(0) + \lambda\phi(0) = 0; \tag{3a}$$

$$\phi(1) = \phi''(1) = 0; \tag{3b}$$

3. ODE SOLVER SOLUTION TECHNIQUES FOR THE EIGENVALUE PROBLEM

We note that Eqs (1) and (3) contain the unknown eigenvalue which must be determined together with the unknown function $\phi(\varphi)$. This task is accomplished by the ODE solver using trivial and equivalent ODE techniques:

$$\lambda' = 0; \tag{4}$$

and

$$R' = \phi^2(\xi); \tag{5a}$$

$$R(0) = 0; \tag{5b}$$

$$R(1) = 1 \tag{5c}$$

which come from the following normalized condition of instability

$$\int_0^1 \phi^2(\xi)\, d\xi = 1 \qquad 0 \le \xi \le 1. \tag{6}$$

While the available general purpose ODE solvers are powerful tools for general linear and nonlinear ODEs, they are not equipped with eigenvalue solution features. In this paper, a solution strategy for structural eigenproblems in ODEs by using standard ODE solvers is presented. First, iteration method of reciprocal power is taken to eliminate other-order eigenfunction vectors with the exception of a dominant vector from initial eigenfunction vectors. Then, the solutions are given by nonlinear Newton iteration method. In the course of iteration of reciprocal power, the technique of eigenvalue displacement is used to accelerate the speed of convergence.

3.1 Iteration method of reciprocal power

A general eigenvalue problem can all be expressed in ODE form as

$$[L]\,\{u(x)\} = \lambda\,[M]\,\{u(x)\} \qquad a \le x \le b, \tag{7}$$

and be subjected to the following boundary conditions:

$$[B_a]\{u(a)\}=\{0\};\ [B_b]\{u(b)\}=\{0\} \tag{8}$$

in which λ is the eigenvalue; {u(x)} is the corresponding eigenfunction vectors of dimensions **n**; [L], [M], [B_a] and [B_b] are all the matrix of linear and differential operators, respectively, and [L] is satisfied with self-adjoint property.

After both the sides of Eq. (7) premultiplied by $[L]^{-1}$, and upon continuing the calculations through **K-1** cycles of iteration, we have

$$\{u(x)\}^{(k)}=\lambda^{(k)}[L]^{-1}[M]\{u(x)\}^{(k-1)}; \tag{9}$$

Letting

$$\lambda^{(k)}=1/\left\|[\tilde{u}(x)^{(k)}\right\|_m; \tag{10}$$

$$\{u(x)\}^{(k)}=\{\tilde{u}(x)\}^{(k)}/\left\|\{\tilde{u}(x)\}^{(k)}\right\|_m; \tag{11}$$

$$\{u(x)\}^{(0)}=\{\tilde{u}(x)\}, \tag{12}$$

and substituting the Eqs (10)-(12) into Eq. (9), resnlts in

$$\{\tilde{u}(x)\}^{(k)}=[L]^{-1}[M]\{u(x)\}^{(k-1)} \tag{13}$$

where $\{\tilde{u}(x)\}$ is the solution of the following linear ODE systems

$$[L]\{\tilde{u}(x)\}^{(k)}=[M]\{u(x)\}^{(k-1)} \qquad a\le x\le b; \tag{14a}$$

$$[B_a]\{\tilde{u}(a)\}^{(k)}=\{0\};[B_b]\{\tilde{u}(b)\}^{(k)}=\{0\}; \tag{14b}$$

Since a reciprocal of the largest eigenvalue of the norm of $[L]^{-1}$ is just the lowest eigenvalue of the norm of [L], the iteration method of reciprocal power is best applied to amplify the lower modes and sweep the higher modes. As the increase of the number of iteration, $\{u(x)\}^{(k)}$ converges to the eigenfunction vector of the lowest order of $\{u_1(x)\}$ involving in the eigenfunction vector of $\{\tilde{u}(x)\}^{(k)}$, and $\lambda^{(k)}$converges to λ_1. So, contributions of the higher mode to the total response are omitted. Then, the filtered vectors are taken as an initial solution. To speed convergence and agree with a predetermined level of accuracy, a nonlinear Newton iteration method is used in the next step for producing a numerical approximation to the solution of the boundary value problem.

3.2 Orthogonalization

To assure **i**th mode to become dominant, **i-1** modes before **i**th mode should be suppressed in assumed initial solution $\{\tilde{u}(x)\}$ by orthogonalization. The orthogonalization process will be applied repeatedly in each iteration step of reciprocal power so that the process results are reliable. In the beginning of iteration of reciprocal power, the initial vectors are assumed as

$$\{u_i\ (x)\}^{(0)} = \{\tilde{u}_i\ (x)\} - \sum_{j=1}^{i-1} \alpha_j\ \{u_j\ (X)\} \tag{15}$$

in which

$$\alpha_j = \frac{1}{b-a} \int_a^b \{u_j(X)\} T[M]\ \{\tilde{u}_j(X)\}\, dx, \quad 1 \le j \le i-1; \tag{16}$$

3.3 Technique of eigenvalue displacement

If an eigenvalue of λ_1 is very close to another eigenvalue of λ_2, in the course of iteration of reciprocal power, the speed of convergence will be very slow. Therefore, for the purpose of aiding convergence, the technique of eigenvalue displacement is also available because their eigenfunction vectors do not change. The ODE system after the displacement is

$$[L^*]\{u(X)\} = \lambda^*[M]\{u(X)\} \tag{17}$$

in which

$$[L^*] = [L] - \mu[M]; \tag{18}$$

$$\lambda^* = \lambda - \mu \tag{19}$$

where μ is the value of eigenvalue displacement.

For reliability and simplicity, in this paper, it is advantageous to speed its convergence using a simple artifice that will be expressed as follows:

$$\mu_i = 0.99\mu_{i-1}. \tag{20}$$

4. NUMERICAL EXAMPLE

Numerical example: Coupled shear-wall building subjected to a distributed axial load; The initial input solution required by the ODE solver–COLSYS is (see references [2 . 3])

$$\phi = z\ (H - z)(1 + z + z^2 + z^3) \tag{21}$$

Other data used by the ODE solver are

Tol (tolerance) = 0.00005;

Ncolp (number of Gauss's collocation point in each subregion)=5;

Nsubi (number of subregion in initial collocation network)=1;

Table 1 shows the results and the relationship between λ and α H given by the solver. To assess the accuracy of the solution, some results obtained by the method of weighted residuals[1] are compared in the table too.

Table 1. Coupled shear-wall critical load parameter of λ by different methods (γ=0/25)

Method	αH=1	αH=3	αH=6	αH=9	αH=14	αH=50	αH=100
Solver	10.510	21.088	30.472	34.462	37.007	39.002	39.140
W.R.M %	10.511 (0.10)	21.100 (0.06)	30.485 (0.04)	34.470 (0.02)	37.012 (0.01)	39.004 (.005)	39.141 (.003)

5. CONCLUSIONS

Table 1 shows that the results given by the present solver are very close to the ones obtained by using method of weighted residuals. Excellent convergence and high accuracy results were obtained in all computed cases (αH=1-100), which show that the advantages of availability and reliability of the existing ODE. This solver can conveniently be taken for accurate and reliable solution of ODE eigenvalue problems, and establish the attractiveness of the present semi-discrete approach to a class of structural eigenvalue problems.

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